

THE STATUS OF THE Λ TERM IN QUANTUM GEOMETRODYNAMICS IN EXTENDED PHASE SPACE

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Abstract

S. Weinberg pointed out a way to introduce a cosmological term by modifying the theory of gravity. This modification would be justified if the Einstein equations with the cosmological term could be obtained in the classical limit of some physically satisfied quantum theory of gravity. We propose to consider quantum geometrodynamics in extended phase space as a candidate for such a theory. Quantum geometrodynamics in extended phase space aims at giving a selfconsistent description of the integrated system “the physical object (the Universe) + observation means”, observation means being represented by a reference frame. The Λ term appears in classical equations under certain gauge conditions and characterizes the state of gravitational vacuum related to a chosen reference frame. The eigenvalue spectrum of Λ depends on a concrete cosmological model and can be found by solving the Schrödinger equation for a wave function of the Universe. The proposed version of quantum geometrodynamics enables one to make predictions concerning probable values of the Λ term at various stages of cosmological evolution.

1 Introduction

Among various approaches to solving the cosmological constant problem, S. Weinberg [1] pointed to the modification of the theory of gravity when the cosmological term would appear in the Einstein equations as an integration constant. Weinberg’s viewpoint was that this modification would be justified if the Einstein equations with the cosmological term could be obtained in the classical limit of some physically satisfied quantum theory of gravity.

In this paper I shall consider quantum geometrodynamics (QGD) in extended phase space as a candidate for such a theory. The proposed modification of QGD enables one to make predictions concerning probable values of the Λ term at various stages of cosmological evolution.

However, this modification is gauge-noninvariant in general. So the question arises, what grounds do we have for considering a gauge-noninvariant quantum geometrodynamics?

2 The grounds for considering a gauge-noninvariant version of quantum geometrodynamics

Now I shall try to show that we do really have such grounds when constructing quantum theory of a closed universe. One of the reasons is that there are no asymptotic states in a closed universe. Meanwhile, any gauge-invariant quantum field theory is essentially based on *the assumption about asymptotic states*. In the path integral approach, which is more adequate for quantizing a gauge theory, asymptotic boundary conditions ensure gauge invariance of a path integral and plays the role of selection rules [2]. However, in a general case without asymptotic states it is mathematically impracticable to separate physical degrees of freedom from “nonphysical” ones and identify gravitational field with a system with two field degrees of freedom.

Another argument is the well-known parametrization noninvariance of the Wheeler – DeWitt equation (see, for example [3, 4]). However, a transition to another gauge variable is formally equivalent to imposing a new gauge condition, and vice versa. The latter reflects an obvious fact that the choice of gauge variables and the choice of gauge conditions have a unified interpretation: they together determine equations for the metric components $g_{0\mu}$, fixing a reference frame.

$$\begin{array}{lll} \text{Parametrization} & + & \text{Gauge conditions} \Rightarrow \text{Equations for } g_{0\mu} \\ g_{0\mu} = v_{\mu}(\mu_{\nu}, \gamma_{ij}) & \mu_{\nu} = f_{\nu}(\gamma_{ij}) & g_{0\mu} = v_{\mu}(f_{\nu}(\gamma_{ij}), \gamma_{ij}) \end{array}$$

Here μ_{ν} are new gauge variables, in particular, the lapse and shift functions, N and N_i , γ_{ij} is 3-metric. Thus even if one considers μ_{ν} as independent of γ_{ij} , different parametrizations will correspond to different reference frames. So, as a matter of fact, *the parametrisation noninvariance of the Wheeler – DeWitt equation is ill-hidden gauge noninvariance*.

3 Quantum geometrodynamics in extended phase space: a minisuperspace example

Bearing in mind all the mentioned above, the investigation of a more general theory seems to be reasonable. In the work by G. M. Vereshkov, V. A. Savchenko and me [5, 6] quantum geometrodynamics in extended phase space has been proposed. The extended phase space (EPS) approach developed by Batalin, Fradkin and Vilkovisky (BFV) [7] – [10] is adequate for studying effects related to gauge degrees of freedom.

The central part in our version of QGD is given to the Schrödinger equation for a wave function of the Universe. The Schrödinger equation is derived from a path integral by the

standard method [11] originated from Feynman. In accordance with the physical situation we consider the path integral without asymptotic boundary conditions. To skeletonize the path integral the full set of equations in EPS is used. This set of equations is gauge-noninvariant and nondegenerate. As a result, no ill-definite mathematical expression arises when deriving the Schrödinger equation, and the procedure of its derivation turns out to be correct but the Schrödinger equation will contain information about parametrization and gauge.

Let us turn to a minisuperspace model which involves isotropic universe and Bianchi IX cases. In a rather broad class of parametrization and gauge conditions the action in EPS can be reduced to the Faddeev – Popov effective action:

$$S_{eff} = \int dt \left\{ \frac{1}{2} \frac{v(Q^a)}{\mu} \gamma_{ab} \dot{Q}^a \dot{Q}^b - \frac{\mu}{v(Q^a)} U(Q^a) + \pi (\dot{\mu} - f_{,a} \dot{Q}^a) - i\mu \dot{\theta} \bar{\theta} \right\}. \quad (1)$$

Here μ is a new gauge variable defined by

$$\frac{N}{a^3} = \frac{\mu}{v(Q^a)}, \quad (2)$$

Q^a are physical variables, a is a scale factor, $\theta, \bar{\theta}$ are the Faddeev – Popov ghosts after replacement $\bar{\theta} \rightarrow -i\bar{\theta}$; π is a Lagrange multiplier, and the special class of gauges not depending on time is used

$$\mu = f(Q^a) + k; \quad k = \text{const}, \quad (3)$$

or, in a differential form,

$$\dot{\mu} = f_{,a} \dot{Q}^a, \quad f_{,a} \stackrel{\text{def}}{=} \frac{\partial f}{\partial Q^a}; \quad (4)$$

$$\gamma_{ab} = \text{diag}(-1, 1, 1, 1, \dots). \quad (5)$$

The Schrödinger equation for this model reads

$$i \frac{\partial \Psi(Q^a, \mu, \theta, \bar{\theta}; t)}{\partial t} = H \Psi(Q^a, \mu, \theta, \bar{\theta}; t), \quad (6)$$

where

$$H = \frac{i}{\mu} \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} - \frac{1}{2M} \frac{\partial}{\partial Q^\alpha} M G^{\alpha\beta} \frac{\partial}{\partial Q^\beta} + \frac{\mu}{v(Q^a)} (U + V); \quad (7)$$

M is the measure in the path integral,

$$M = v^{\frac{K}{2}}(Q^a) \mu^{1-\frac{K}{2}}; \quad (8)$$

$$G^{\alpha\beta} = \frac{\mu}{v(Q^a)} \begin{pmatrix} f_{,a} f^{,a} & f^{,a} \\ f^{,a} & \gamma^{ab} \end{pmatrix}; \quad \alpha, \beta = (0, a); \quad Q^0 = \mu, \quad (9)$$

K is a number of physical degrees of freedom; the wave function is defined on extended configurational space with the coordinates $Q^a, \mu, \theta, \bar{\theta}$. The feature of the present approach is the

appearance of a quantum correction V to the potential U . The correction is due to the dependence of the metric of configurational space of physical variables, $G_{ab}^{(phys)} = \frac{v(Q^a)}{\mu} \gamma_{ab}$, on these variables. V depends on the chosen parametrization and gauge (2), (3):

$$V = -\frac{K^2 + 5K}{24\mu^2} f_{,a} f^{,a} + \frac{K+1}{6\mu} f_{,a}^{,a} + \frac{K^2 - K - 2}{12\mu v(Q^a)} v_{,a} f^{,a} - \frac{K^2 - 7K + 6}{24v^2(Q^a)} v_{,a} v^{,a} + \frac{1-K}{6v(Q^a)} v_{,a}^{,a}. \quad (10)$$

The investigation of the set of equations in EPS reveals the existence of a conserved quantity $E = \mu\dot{\pi}$. As a result, the Hamiltonian constraint $H_{ph} = 0$ of general relativity is replaced by the constraint $H = E$, where H_{ph} is a Hamiltonian of gravitational and matter fields, H is a Hamiltonian in EPS.

The latter means that a Hamiltonian spectrum in the appropriate quantum theory is not limited by the unique zero eigenvalue. Finding a spectrum of E becomes one of the main tasks of quantum geometrodynamics in EPS.

The general solution to the Schrödinger equation (6) has the following structure:

$$\Psi(Q^a, Q^0, \theta, \bar{\theta}; t) = \int \Psi(Q^a) \exp(-iEt) (\bar{\theta} + i\theta) \delta(\mu - f(Q^a) - k) dE dk. \quad (11)$$

The dependence of the wave function (11) on ghosts is determined by the demand of norm positivity. The “physical part” of the wave function $\Psi(Q^a)$ is a solution to the equation

$$H^0 \Psi(Q^a) = E \Psi(Q^a); \quad (12)$$

$$H^0 = \left[-\frac{1}{2M} \frac{\partial}{\partial Q^a} \frac{\mu}{v(Q^a)} M \gamma^{ab} \frac{\partial}{\partial Q^b} + \frac{\mu}{v(Q^a)} (U + V) \right] \Big|_{\mu=f(Q^a)+k}. \quad (13)$$

The wave function (11) carries the information on the physical object (the Universe) and a chosen reference frame, the latter representing the observer in the theory of gravity. The correlations between the properties of the physical object and those of the reference frame are manifested in the quantum correction V to the potential. So quantum geometrodynamics in extended phase space aims at giving a selfconsistent description of the integrated system “the physical object (the Universe) + observation means (a reference frame)”.

The line $E = 0$ in the Hamiltonian spectrum is of a particular interest. If one puts $E = 0$ and chooses the gauge $\dot{\mu} = 0$, the Schrödinger equation for the physical part of the wave function will be reduced to the Wheeler – DeWitt equation written down for an arbitrary parametrization of a gauge variable. As we can see, quantum geometrodynamics in EPS involves the Wheeler – DeWitt QGD as a particular case. The way of derivation of the Wheeler – DeWitt equation demonstrates that the Wheeler – DeWitt QGD is not a gauge-invariant theory in a strict mathematical sence. However, taking a classical limit for the wave function of the state with $E = 0$ one obtains gauge-invariant equations of motion¹.

¹If the line $E = 0$ belongs to a continuous part of the spectrum, the state with $E = 0$ cannot be normalized

4 The cosmological constant problem

Now let us turn to the cosmological constant problem. Consider parametrization and gauge

$$\mu = Na^3; \quad \mu = 1. \quad (14)$$

It corresponds to the constraint on the components of 4-metric

$$\sqrt{-g} = \text{const}. \quad (15)$$

As was shown by Weinberg [1], imposing the condition (15) leads to the appearance of an additional term in the Einstein equations

$$T_{\mu(obs)}^\nu = -\frac{1}{2\pi^2}\Lambda\delta_\mu^\nu, \quad (16)$$

and it follows from the set of equations in EPS that there exist a conserved quantity

$$E = -\int d^3x \sqrt{-g} T_{0(obs)}^0 = \Lambda. \quad (17)$$

The eigenvalue spectrum of Λ can be found by solving the Schrödinger equation; results will depend on a chosen cosmological model.

Taking into account the effect of particle creation in the early Universe, one can consider quantum transisions between states with different values of Λ and make predictions concerning probable values of Λ at various stages of cosmological evolution.

To summarize, in quantum geometrodynamics in extended phase space the Λ term has the following status:

- The Λ term characterizes the state of gravitational vacuum related to a chosen reference frame.
- $\Lambda = 0$ corresponds to the state of the Universe in which gauge effects are negligible; the evolution of the Universe can be described in the classical limit by a gauge-invariant theory;
- Λ may take on nonzero values at an early stage of cosmological evolution, when gauge-noninvariant effects should be taken into account.

and therefore is not physical. To deal with physical states one should consider narrow enough wave packets, so that the mean value of E with respect to such a packet would be equal to zero.

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